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Function Art: Understanding the Artistic Transformation of Mathematical Functions through Learning Analytics

Guillermo Bautista Jr.[1], Roderick Cacuyong[2], Zsolt Lavicza[3], Barbara Sabitzer[4] and Mona Emara[5]

Abstract

This study explores how students across Grades 8 to 12 engage with mathematical functions in creative, visual ways through function art—an innovative STEAM-based educational approach. Grounded in the Trends in International Mathematics and Science Study (TIMSS) framework and employing a Design-Based Research methodology, the project involved 400 students from the Philippines who created digital artworks using GeoGebra. To uncover learner profiles, a person-centred clustering method—hierarchical clustering on principal components—was applied to variables representing the number and types of functions used. The results revealed three distinct student profiles: Repetitivists (high function quantity, low diversity), Simplists (low quantity and diversity), and Multifunctionists (high diversity, low quantity). Further analysis showed meaningful associations between cluster membership, grade level, and function strategies. Qualitative evaluation using TIMSS cognitive domains—Knowing, Applying, and Reasoning—highlighted that students' use of mathematical strategies and precision in transformations varied widely, often independently of the quantity or diversity of functions used. These findings suggest that function art, when analyzed through learning analytics, provides a rich lens for understanding students' mathematical thinking and offers valuable insights for tailoring interdisciplinary instruction in STEAM education.

Notes for Practice

- Function art can reveal distinct student profiles—such as Repetitivists, Simplists, and Multifunctionists—enabling educators to differentiate instruction based on students' mathematical strategies and levels of engagement.
- Teachers should go beyond simply counting function types and instead examine the precision and strategies students use when creating visual representations of functions.
- Integrating TIMSS cognitive domains (Knowing, Applying, Reasoning) into evaluation provides a structured way to assess student work and inform targeted pedagogical interventions in STEAM contexts.

Keywords: Function art, learning analytics, STEAM education, mathematical function, student profiling

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Corresponding author [1]Email: gpbautista1@up.edu.ph Address: UP NISMED, University of the Philippines Diliman, STTC Building, Quirino Ave. cor. Velasquez St., U.P. Diliman, 1101 Quezon City, Philippines; Linz School of Education, Johannes Kepler University Linz, Science Park 5, 4th Floor, Altenberger Straße 68, 4040 Linz, Austria. ORCID iD: https://orcid.org/0000-0001-5471-9326

[2]Email: roderick.cacuyong@deped.gov.ph Address: Isabela National High School, Claravall Street, San Vicente, City of Ilagan, Isabela 3300, Philippines.

[3]Email: zsolt.lavicza@jku.at Address: Department of STEM Education, Linz School of Education, Johannes Kepler University Linz, Science Park 5, 4th Floor, Altenberger Straße 68, 4040 Linz, Austria. ORCID iD: https://orcid.org/0000-0002-3701-5068

[4]Email: barbara.sabitzer@jku.at Address: Department of STEM Education, Linz School of Education, Johannes Kepler University Linz, Science Park 5, 4th Floor, Altenberger Straße 68, 4040 Linz, Austria. ORCID iD: https://orcid.org/0000-0002-1304-6863

[5]Email: mona.emara@dmu.edu.eg Address: Department of Educational Psychology, Faculty of Education, Damanhour University, Alabadya, Damanhour 22516, Egypt; Linz School of Education, Johannes Kepler University Linz, Science Park 5, 4th Floor, Altenberger Straße 68, 4040 Linz, Austria. ORCID iD: https://orcid.org/0000-0002-9277-7073

1. Introduction

Despite longstanding efforts to enhance mathematics instruction, many students continue to face persistent difficulties in understanding and applying function concepts (National Council of Teachers of Mathematics, 2000). These challenges often arise from the abstract nature of functions, where learners must connect symbolic notation, graphical representations, and real-world applications (Troup et al., 2024; Troup, 2015). To address these barriers, innovative educational approaches that integrate hands-on, creative experiences have proven beneficial (Boaler, 2015). In particular, STEAM—an interdisciplinary framework that merges Science, Technology, Engineering, Arts, and Mathematics—has gained traction for fostering engagement and deeper conceptual understanding (Yakman, 2008). By bringing visual arts into the study of mathematics, students can form more tangible and personally meaningful connections to abstract concepts (Freedman, 2003; Marshall, 2014). This arts-integrated perspective not only promotes higher levels of engagement but also cultivates creativity, problem-solving skills, and a sense of ownership over the learning process (Land, 2013).

Building on these insights, our function art approach (Bautista, Prodromou, Raynes et al., 2024; Bautista, Prodromou, & Lavicza, 2024) leverages visual arts to facilitate students' exploration of functions. By designing visual representations and creating artworks, learners illustrate mathematical relationships and transformations in a tangible, personalized manner. This approach encourages students to investigate key ideas, such as domain, range, intervals, and transformations, through hands-on exploration (Bautista, Malacapo et al., 2025). Given that functions constitute a foundational element of higher mathematics, experiences gained through function art may bolster students' conceptual understanding and better prepare them to tackle more advanced topics.

While STEAM-based strategies such as function art are gaining traction, research on their implementation is still emerging, particularly from an analytic standpoint. In parallel, the field of learning analytics has grown substantially in pre-tertiary education, offering new possibilities for examining how students interact with content and learn over time (Kovanovic et al., 2021). However, much of this research remains siloed within traditional academic disciplines, with most studies focusing exclusively on mathematics or art in isolation (Moon et al., 2024). Interdisciplinary contexts, especially those that combine math and art, have been given comparatively little attention.

Moreover, the application of learning analytics to identify student learning profiles in STEAM contexts remains underexplored. Although some studies have used log data to identify general learning patterns, few have examined how distinct cognitive or behavioural profiles have emerged across grade levels in interdisciplinary environments (Aguerrebere et al., 2022; De Sousa et al., 2021). This presents a critical research gap: we lack a nuanced understanding of how students engage with interdisciplinary tasks such as function art, and how their approaches and abilities evolve from Grades 8 through 12.

To address this gap, this study employs a person-centred approach, including cluster analysis, which is well-suited to identifying patterns among subgroups of learners (Asendorpf, 2002; Bergman, 2000). Unlike variable-centred methods that focus on average trends, person-centred techniques allow for a more granular examination of how different students navigate the learning process, enabling educators to tailor instruction to meet diverse needs.

This paper aims to investigate the strategies, methods, and types of functions students use when engaging in function art activities. Specifically, it seeks to identify distinct learner profiles based on their interactions with mathematical and artistic elements and to examine how these profiles are distributed across grade levels. The research is guided by the following questions:

RQ1. What student profiles emerge from function art activities in a dynamic learning environment?

RQ2. How does the distribution of the emerging profiles differ across Grades 8–12?

By addressing these questions, the study contributes to a more comprehensive understanding of interdisciplinary learning in STEAM education. It also advances the use of learning analytics to inform differentiated instruction and curriculum design, offering insights into how creative approaches can support mathematical thinking across developmental stages.

2. Background and Literature

In this section, we discuss the previous studies on STEAM education and types of STEAM integration, function art, and the conceptual framework of this study.

2.1. Integrating STEM and ART: From STEM to STEAM

The shift from STEM (Science, Technology, Engineering, and Mathematics) to STEAM reflects a growing recognition that artistic thinking and creativity contribute essential cognitive, affective, and sociocultural dimensions to disciplinary learning. Artistic modes of inquiry—imagination, critique, and meaning-making—extend beyond embellishment to become integral to how learners reason, express, and construct understanding (Henriksen, 2017; Costantino, 2018; Halverson & Sawyer, 2022). Within this framework, mathematics learning through art aligns with a constructivist and design-based tradition that positions learners as active creators of knowledge. From Papert's (1980) constructionism to contemporary digital design environments such as GeoGebra or Desmos, mathematical expression through artistic creation allows students to treat functions and equations as materials for composition—transforming mathematics from a static system of rules into a dynamic medium for exploration, interpretation, and personal meaning (Dietiker, 2015; Bautista, Malacapo et al., 2025). Integrating the arts into mathematics education also draws on the cognitive and epistemic potential of creative practice. Arias-Alfonso and

Franco (2021) describe the creative act in mathematical art as simultaneously logical and emotional, engaging reasoning and intuition to generate new knowledge. Digital tools make this dual engagement visible: when students use graphing technologies to "draw" with functions, they externalize complex mathematical ideas in ways that promote abstraction, visualization, and reflection (Fenyvesi et al., 2019; Portaankorva-Koivisto & Havinga, 2019). Such activities embody what Perales and Aróstegui (2024) term the cognitive and sociocultural integration of STEAM, in which learning is both intellectually rigorous and socially situated. They argue that STEAM reframes education from content-centred instruction toward learner-centred inquiry that values creativity, collaboration, and civic participation. Correspondingly, Kim and Kim (2016) identify four domains of STEAM competence—cognitive, higher-order thinking, communal, and emotional—each of which is activated in function-based art-making as students reason mathematically, design aesthetically, work collaboratively, and express personal meaning.

Critically, as Mejias et al. (2021) argue, the integration of arts and STEM should move beyond superficial or instrumental models, where art merely serves to illustrate or "sugarcoat" technical content, toward a mutually instrumental model in which both disciplines are "equally in play." Such integration, they contend, can surface productive "epistemic friction": the tension between disciplinary norms and practices (e.g., precision versus ambiguity, rule-following versus open interpretation) that prompts deeper inquiry and reflection. In this light, function art tasks invite learners to navigate and negotiate across disciplinary boundaries, balancing mathematical exactness with artistic expression, thereby supporting more holistic forms of learning.

Beyond cognition, arts education contributes socio-emotional and communicative practices fundamental to learning. Perignat and Katz-Buonincontro (2019) emphasize critique, self-expression, and the conveyance of meaning as hallmark features that cultivate empathy, reflection, and openness to multiple perspectives—capacities essential for 21st-century citizenship. As Perales and Aróstegui (2024) note, STEAM thus helps reconcile the longstanding debate between art-as-means and art-as-end by acknowledging the intrinsic value of aesthetic engagement while leveraging it for cognitive and social growth. In this sense, STEAM education embodies what Bevan et al. (2020) call "art-science": a purposeful space where students move beyond fixed identities as "mathsy" or "artsy" and instead operate as flexible, integrative thinkers.

Empirically, research in mathematics education shows that such integration strengthens conceptual understanding without sacrificing rigour. Wilkie (2024) found that figural-pattern tasks fostered connections among multiple representations of functions and supported deeper algebraic reasoning. Schoevers et al. (2020) demonstrated that students engaged in visual-arts-geometry activities achieved equal or superior performance on standardized assessments while exhibiting heightened spatial reasoning.

Together, these theoretical and practical insights indicate that STEAM approaches in mathematics, especially those involving visual, hands-on, and creative components, can support both conceptual learning and student engagement. Importantly, these outcomes appear particularly strong in open-ended or exploratory tasks, such as function art, where students must apply mathematical principles to construct personally meaningful artifacts (Berland et al., 2014). However, the diversity of student approaches in such tasks—varying in strategy, fluency, and use of function types—points to the need for structured analytic tools to better understand how different learners engage with these tasks (Oviatt et al., 2018).

2.2. Learning Analytics to Understand Behaviour

In parallel with the growing interest in STEAM, Learning Analytics (LA) has emerged as a powerful approach to capturing, analyzing, and interpreting data generated by learners in technology-enhanced environments (Ifenthaler & Yau, 2020; Siemens, 2013). LA has proven instrumental in deciphering the complexities of learner behaviours and strategies, particularly through trace data. Trace data analysis in LA traditionally bifurcates into two main research strands: predictive modelling and learner profiling (Whitehill et al., 2017; Xing et al., 2016). Predictive models focus on forecasting student success or identifying at-risk students by analyzing engagement metrics such as time spent on resources and the number of active learning days (Caspari-Sadeghi, 2022; Gardner & Brooks, 2018). However, another equally critical strand involves using cluster analysis to profile learners based on their learning or problem-solving patterns across various activities (Cobo Rodríguez et al., 2011; Khalil & Ebner, 2017; Emara et al., 2025).

In the present study, cluster analysis is framed not as a neutral categorization tool but as a theoretically informed method aligned with constructivist and cognitivist views of learning (Papert, 1980; Dietiker, 2015; Mejias et al., 2021). These perspectives emphasize that learning is dynamic, contextually situated, and often expressed through diverse forms of reasoning, representation, and creativity. Cluster analysis thus becomes a means of surfacing learner diversity—specifically how students use mathematical functions to construct visual representations within a function art task.

This methodological stance supports a shift from purely outcome-based evaluation to an emphasis on process, aligning with broader calls in the field to understand how learning unfolds in authentic, interdisciplinary settings (Knight et al., 2017; Alhadad, 2018). As noted in the learning sciences literature (e.g., Bouchet et al., 2013; Kovanović et al., 2019), clustering can reveal latent patterns in learner behaviour, especially in open-ended tasks where traditional performance metrics are insufficient. In our case, the resulting profiles provide insight into the different ways students engage with mathematics in a creative setting.

By combining theoretical insights from STEAM education with methodological tools from LA, this study contributes a novel process-oriented perspective on how students engage with mathematics through artwork practice. The integration of cluster analysis not only deepens our understanding of learner diversity within STEAM environments but also offers a scalable method for researchers and educators to investigate learning at the intersection of computation, design, and mathematical thinking. As we argue in this study, clustering techniques provide one such method for identifying distinct learner profiles within creative mathematical contexts, offering insights into how students reason, explore, and construct meaning through STEAM-based activities.

2.3. Function Art in Educational Context

The use of functions in the creation of artworks has been explored by educators since the early 90s. However, because of the scarcity of graphing calculators at the time, most of these studies used paper and pencil (e.g., Avila, 2013; Disher, 1995). This approach has several limitations: first, it restricts the number of functions that students use (Zembat, 2008); second, it makes it difficult to derive algebraic equations from geometric curves, especially when the curves are complex or irregular in shape (Coskun, 2011; Kieran & Drijvers, 2006); and third, it narrows the types of functions that can be explored (Komatsu & Jones, 2020).

With the advent of graphing calculators and later dynamic graphing software, the pedagogical landscape changed significantly. Graphs could be rendered instantly by entering function equations, enabling students to quickly test and manipulate parameters, explore a broader range of function families, and engage in more iterative, exploratory learning (Bautista, Malacapo et al., 2025). These technological affordances also made it easier to move fluidly between algebraic and geometric representations—a key goal in function learning.

Despite this advancement, most early work in function art remained largely descriptive and exploratory. For example, Disher (1995) had students draw figures on graph paper and then identify the functions that best modelled each curve. Similar paper-based methods continued to be used in subsequent studies (Black, 2011; Lee, 2002), while Avila (2013) replicated Disher's approach using graphing calculators to aid equation generation. Yet, across these studies, little attention was paid to how students' strategies varied or how their mathematical choices reflected deeper conceptual reasoning.

More recent projects have expanded the creative potential of function art while reinforcing its educational value. Sharp (2007) had students overlay function graphs onto digital images to explore slope, ratio, and symmetry. Barry (2021) investigated biological forms by tracing animal outlines with functions, while Rebholz (2017) engaged students in designing letters of the alphabet using graphs, thereby promoting spatial reasoning and function composition. These studies demonstrate the potential of function art to bridge abstract mathematical concepts with visual, tangible outcomes. However, they focus primarily on instructional application and do not examine how students' individual approaches—such as their selection of function types or sequencing of transformations—might differ or cluster into identifiable patterns.

This variation in student behaviour during open-ended, creative tasks is pedagogically significant. In function art, students make numerous decisions about which functions to use, how to manipulate them, and how to structure their compositions. These decisions can reflect differences in conceptual understanding, comfort with transformations, creativity, and engagement (Papert, 1980; Portaankorva-Koivisto & Havinga, 2019; Bautista, Promdromou, Raynes et al., 2024). Understanding such differences not only reveals learning processes but can also inform differentiated instruction.

To systematically capture this diversity, a person-oriented approach using clustering techniques offers a promising analytic path. Clustering allows researchers to identify distinct learner profiles based on students' functional and artistic strategies, providing insight into how students engage with mathematical tasks—not merely their performance outcomes (Antonenko et al., 2012; Lee et al., 2022). For example, cluster analysis can distinguish between students who predominantly rely on basic linear functions and those who explore complex compositions or use piecewise and trigonometric functions to achieve aesthetic goals. These profiles support a process-oriented understanding of learning and align with the broader goals of LA (Buckingham Shum & Ferguson, 2012).

Building on earlier conceptualizations, function art can be categorized into three types: pure, organic, and hybrid (Bautista, Prodromou, & Lavicza, 2024). *Pure function art* consists solely of function graphs (e.g., linear, quadratic, sine). *Organic function art* primarily uses functions but incorporates mathematical objects, such as conics (e.g., circles, ellipses), when needed. *Hybrid function art* blends functions with geometric primitives, image overlays, and annotations. Figure 1 illustrates these distinctions, highlighting the visual and mathematical diversity across student creations.

The present study extends prior research by incorporating a larger, more diverse sample of students across six schools and five grade levels (Grades 8–12), expanding beyond the Grade 11 focus in earlier work (Bautista, Malacapo et al., 2025; Bautista, Soeharto et al., 2025). By applying clustering techniques, we aim to identify patterns in how students engage with function art and to contribute a theoretically grounded, empirically supported analysis of how student profiles vary within dynamic learning environments.

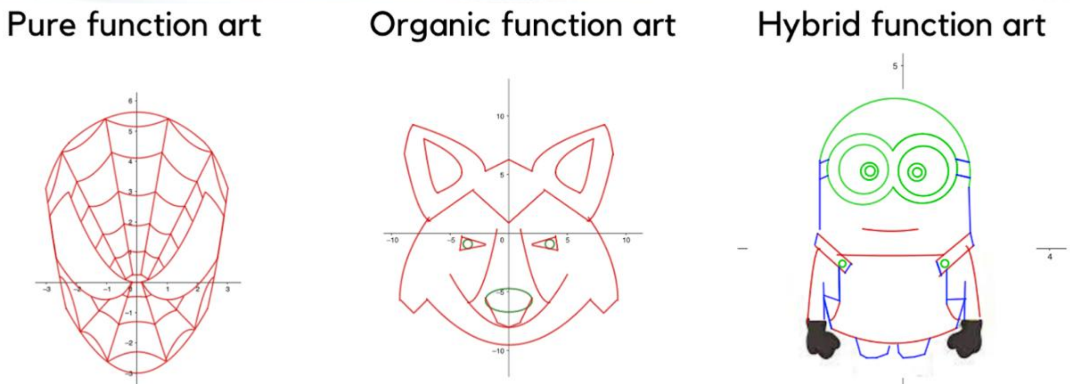

Figure 1. Types of Function Art

Note: Reproduced with permission from Bautista, Prodromou, & Lavicza, 2024.

2.4. Using TIMSS to Identify Cognitive Strategies in Function Art

The Trends in International Mathematics and Science Study (TIMSS) is one of the most comprehensive international assessments of mathematics and science education, conducted every four years since 1995. It evaluates student performance in Grades 4, 8, and 12 across diverse educational systems—72 in the most recent 2023 cycle (von Davier et al., 2024). Beyond its role in benchmarking academic achievement, TIMSS provides a robust framework for analyzing mathematical problem solving, structured around both content and cognitive domains (Mullis et al., 2021).

The TIMSS cognitive domain comprises Knowing, Applying, and Reasoning. These domains are designed to capture not only what students know, but also how they use and extend their knowledge. The Knowing domain assesses recall and recognition of facts, concepts, and procedures. Applying refers to students' capacity to use this knowledge in concrete and routine problem-solving contexts. Reasoning evaluates higher-order thinking, including analysis, synthesis, and evaluation of mathematical situations. For more details about other skills, refer to Mullis et al. (2021).

Despite the comprehensive nature of the TIMSS cognitive framework, its application beyond test construction and content alignment has been limited. Most research using TIMSS focuses on national curriculum alignment or large-scale performance comparisons (Zhai & Pellegrino, 2023). However, a few emerging studies demonstrate the framework's potential for cognitive diagnostic purposes. For example, Wu et al. (2025) used a cognitive diagnostic Q-matrix based on TIMSS 2015 items to assess Chinese 8th-grade students' mastery of cognitive attributes such as Representation Modelling and Analysis & Evaluation. Similarly, Adak (2021) applied the TIMSS cognitive domain categories—Knowing, Applying, and Reasoning—to classify geometry problems and analyze the strategies Turkish students used to solve and justify their responses. Although these studies illustrate the framework's capacity to structure analyses of student cognition, its application to understanding students' cognitive strategies across grade levels in integrated STEM-art tasks—such as function art—remains largely underdeveloped. Function art, which integrates mathematics, technology, and visual arts by graphing mathematical functions, offers a unique avenue for exploring how students employ various cognitive strategies.

In function art tasks, students use mathematical functions to create artwork, often using graphing technology to manipulate parameters and explore symmetry, transformation, and composition. This context creates a unique opportunity to observe cognitive strategies through an artistic and technological lens. Within this framework (as shown in Table 1), Knowing includes recognizing function types (e.g., identifying a parabola as a quadratic function) and retrieving key features from graphs (e.g., endpoints, intercepts). For example, when students recognize a graph's curve as parabolic and enter a quadratic equation accordingly, they are demonstrating recall. Similarly, identifying where to truncate a function based on its domain illustrates knowledge retrieval.

Applying encompasses actions such as adjusting parameters to modify curve shapes (Represent/Model) and implementing appropriate graphing techniques (Implement). A practical illustration is when students use the absolute value function to generate symmetrical shapes instead of manually plotting mirrored lines—demonstrating efficient and purposeful application of mathematical properties.

Reasoning in function art emerges when students integrate multiple representations or creatively evaluate alternative strategies. This might include using symbolic notation (e.g., $f(x)$ and $f(-x)$) to reflect functions across the *y-axis* (Integrate) or dividing a circle into semicircles to satisfy the definition of a function (Evaluate). These behaviours illustrate how students move beyond procedural knowledge to engage in complex problem solving and abstraction.

The TIMSS cognitive domains were selected as an initial analytical lens due to their broad recognition and structured classification of mathematical thinking. While originally designed for standardized assessments, these domains offer a useful scaffold to explore how students interact cognitively with mathematical functions.

Table 1. The TIMSS Cognitive Domains and Function Art Skills

Cognitive Domain	Skills	Definition
Knowing	Recall	Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
	Retrieve	Retrieve information from graphs, tables, texts, or other sources.
Applying	Represent/Model	Display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship.
	Implement	Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures.
Reasoning	Integrate/Synthesize	Link different elements of knowledge, related representations, and procedures to solve problems.
	Evaluate	Evaluate alternative problem-solving strategies and solutions.

This study thus proposes that repurposing the TIMSS cognitive framework for such analysis holds significant promise, offering a structured way to capture both the procedural and strategic dimensions of student problem-solving in interdisciplinary settings.

This study is guided by three main ideas. First, we categorize student-created function art into three types—pure, organic, and hybrid—based on how mathematical functions are used in artistic expression. Second, we draw on STEAM education literature to explore the integration of mathematics, technology, and artistic creativity. Finally, we analyze how students engage with mathematical functions through visual and interactive representations using GeoGebra. Together, these ideas guide our research design and analysis of student work.

3. Methods

This study is guided by Design-Based Research (DBR) to investigate the students' use of functions. DBR is an iterative research methodology aimed at improving products and processes, and at testing new resources and approaches in educational settings (McKenney & Reeves, 2013; Tinoca et al., 2022). In this study, the process is the implementation of function art; the products are the artworks and students' learning outcomes. The resources are the support materials for creating function art. This research is designed to have three iterations. The data in this paper were collected from the first two iterations. The first iteration focused on the mathematical concepts students learned through creating function art. The second iteration explored how to assess the students' artworks. The third iteration investigated integrating function art with non-STEAM subjects. For each iteration, the implementation structure and accompanying support materials were refined based on prior findings. In each cycle, a webinar was held, followed by a period in which students created their artworks.

3.1. Context of the Study

The intervention comprised a single 2.5-hour online webinar delivered at each participating school via video conferencing software. The intervention was conducted online due to logistical constraints, including the geographic distance between the researcher and the study site and the lack of funding for in-person fieldwork. Separate webinars were conducted for Grades 8, 9, and 10, while a combined session was held for Grades 11 and 12, as they share the same prerequisite knowledge. The session focused on GeoGebra, a free dynamic mathematics software that integrates algebra, geometry, calculus, and spreadsheets (Hohenwarter et al., 2008). The webinar included an introduction to GeoGebra's graphical user interface, a tutorial on plotting graphs, and a review of function transformations—specifically, horizontal and vertical translations and dilations. Participants also engaged in activities involving defining function domains over specific intervals and observed a demonstration of creating sample function-based art using the software. The types of functions introduced during the session varied by grade level, aligning with each group's prior curricular exposure to functions.

After the webinar, students were given one month to create their artwork in GeoGebra. Instructions provided to students across grade levels directed them to use GeoGebra to create artwork incorporating mathematical functions. They were encouraged to use as many functions as possible and given the freedom to explore and use a wide range of tools. Throughout this period, teachers were readily available to support students and address any inquiries they had. In addition, support materials such as the PowerPoint presentation used in the webinar and a GeoGebra Book containing interactive versions of the slides were distributed to students. An 18-minute recorded video about creating a simple artwork was also included in the support materials.

3.2. The GeoGebra Environment

GeoGebra is a dynamic mathematics software that supports multiple representations (Bautista et al., 2023). Its environment, as shown in Figure 2, includes the Graphics view, Algebra view, and Construction Protocol, which we used to collect data and analyze students' artwork. The Graphics view displays the geometric representations of mathematical objects, such as points and graphs. The Algebra view contains the algebraic representations of these objects, such as coordinates and equations. The Construction Protocol shows the chronological order in which the objects were created.

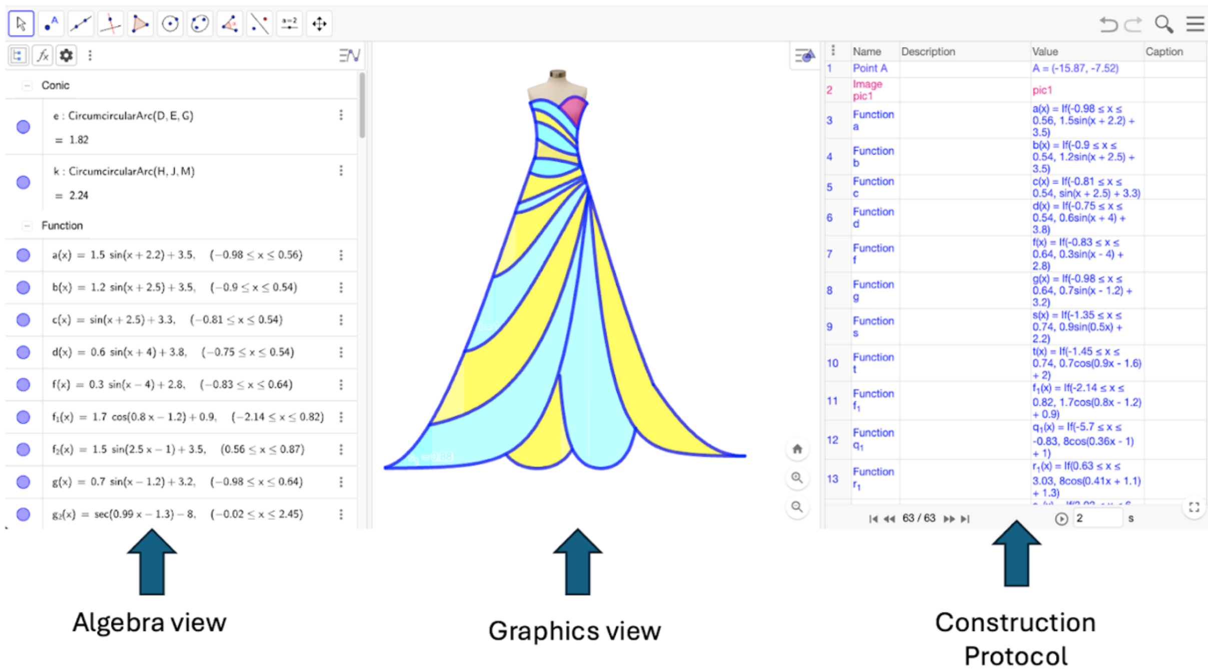

Figure 2. The GeoGebra Interface

The objects in the Algebra view can be grouped by dependency, object type, layer, and construction order. This provided us the flexibility to examine the artworks. For instance, grouping objects by type (see Figure 2, left pane) makes it easy to count the functions, as they are clustered together. Meanwhile, the Construction Protocol lets you see the step-by-step construction by clicking the play and back buttons (see bottom of right pane in Figure 2).

3.3. Data Collection

A convenience sampling technique (Patton, 2014) was used to select 400 students from Grades 8 to 12 (144 males, 256 females) across six schools in the Philippines. Participants were selected from schools in which the teachers involved in the project were familiar with the first author. This method was necessary due to difficulties in recruiting schools, as many teachers were hesitant to participate, and access to technology and the internet was limited. Additionally, because entire classes were selected at each school, we had no control over the gender distribution of participants. We did not collect the students' GPAs because of privacy concerns. Grade 11 students already had prior knowledge of GeoGebra, but we trained everyone in the prerequisite skills before starting the study.

We ensured a diverse mix of schools, including students from various socioeconomic backgrounds. School 1 predominantly consisted of students from middle-class families, all of whom had access to multiple devices and uninterrupted internet connectivity. School 6 comprised students from under-resourced backgrounds who primarily relied on mobile phones and often lacked home internet access. The other schools represented a blend of socioeconomic backgrounds.

After completing their artwork, students uploaded it to their GeoGebra account and submitted the URL via Google Form. Initially, we collected a total of 409 artworks. Each student was identified by a unique student ID, and we also collected their gender, grade level, and corresponding GeoGebra function art. To ensure the quality of the dataset, we removed outliers—defined as artworks with a number of functions exceeding two standard deviations from the mean—resulting in a final sample of 400 artworks.

This study was approved by the Ethics Review Committee of the University of Mindanao, in accordance with the ethical guidelines for research involving human participants in the Philippines. The approval was granted under protocol number UMERC2025122. Informed consent was obtained from the parents or legal guardians of all participating students, and student assent was secured before data collection.

3.4. Data Analysis

A convergent mixed-methods design (Creswell & Creswell, 2018) was employed to comprehensively investigate how students utilized mathematical functions in their artwork. Quantitatively, cluster analysis (Bergman, 2000) was used to identify distinct profiles of students' strategic approaches, based on the number and types of functions used. For the qualitative component, representative samples from each cluster were analyzed using the TIMSS cognitive domains framework to categorize and interpret students' mathematical thinking.

3.4.1. Coding Procedure

We conducted content analysis and applied a systematic coding procedure inspired by Krippendorff (2019) to tally the number of functions by function type and the total number of functions per artwork (see Table 2 for coding criteria). Two coders used GeoGebra's Algebra View, which displays the function equations, to tally the number of functions by function type. Function types were strategically categorized taking into account various factors, including their mathematical properties, alignment with the curriculum, and insights derived from the literature review (e.g., Bautista, Malacapo et al., 2025; Stewart et al., 2024). This resulted in nine function types: (1) constant, (2) linear, (3) absolute value (abs), (4) quadratic, (5) polynomials with degree $n \geq 3$, (6) radical functions, (7) sine/cosine functions, (8) exponential/logarithmic functions, and (9) others. Constant and linear functions were distinguished from each other due to the potential link between constant functions and challenges in understanding the concept of range (Cho & Moore-Russo, 2014).

To ensure the accuracy and consistency of our coding process, we employed Cohen's Kappa, yielding a Kappa value of 0.95, indicating a substantial degree of agreement between coders. All disagreements were resolved, benefiting from the clear and explicit classifications of functions. This approach allowed us to analyze the data with a high level of confidence in the agreement between the coders, further strengthening the reliability of our findings. While coding, we noted artworks that solely used function graphs, which are called pure function art.

After tallying the number of functions used in each artwork, we classified each artwork into one of three function art types: pure, organic, or hybrid (Bautista, Prodromou, & Lavicza, 2024). Since pure function art had already been identified, the remaining task was to distinguish between organic and hybrid function art. The classification algorithm was based on a guiding principle: if a student used functions whenever possible, the artwork was considered organic; otherwise, it was classified as hybrid. In other words, hybrid artworks contain curves or shapes that could have been achieved with functions, but the student chose not to use them. The specific classification rules were as follows:

Artworks that used only functions and circles were classified as organic, since representing a circle algebraically (e.g., as two semicircles or arcs) is not straightforward in GeoGebra.

Artworks that used only functions and vertical line segments were also classified as organic, because vertical segments cannot be represented as functions.

Artworks that included arcs were classified as hybrid if the arcs passed the vertical line test; otherwise, they were classified as organic.

Artworks that used functions along with other geometric objects not covered above (e.g., general line segments, polygons, or imported images) were classified as hybrid.

To facilitate the easy identification of geometric objects, we adjusted their visual attributes in GeoGebra. For example, changing the colour of all line segments to bright colours helped us quickly determine whether they were vertical.

3.4.2. Cluster Analysis

This study employed hierarchical clustering on principal components (HCPC) (Audigier et al., 2016), implemented through the FactoMineR and FactoShiny packages in R to identify distinct profiles of students' function art strategies. HCPC analysis combines principal component analysis (PCA), hierarchical clustering (using Ward's linkage and squared Euclidean distances), and k-means clustering, thereby reducing dimensionality, eliminating noise, and enhancing the robustness of the clustering solution (Ding & He, 2004; Landau et al., 2011; Xu et al., 2015).

The initial PCA step retained the dimensions that explained the greatest variance in the dataset. These dimensions were derived from two theoretically grounded clustering features: (1) the number of functions used, and (2) function diversity. These features reflect key constructs within cognitive constructivism. Specifically, function count operationalizes students' iterative construction of knowledge through repeated cycles of experimentation and refinement, while function diversity captures variation in representational strategies, a hallmark of conceptual flexibility and development (Clements & Battista, 1992; Papert, 1991). In transdisciplinary STEAM settings, these observable features represent more than procedural behaviour—they indicate learners' agency, conceptual progression, and the co-construction of mathematical meaning through aesthetic expression (Mejias et al., 2021; Costantino, 2018).

Next, hierarchical clustering was used to minimize intra-group variance, and the cluster solution was refined using a k-means procedure. To confirm the stability of this solution, cross-validation was performed by randomly splitting the dataset (70% training, 30% test). Each subset was analyzed separately. Silhouette analysis (range: -1 to +1) was then used to evaluate the internal validity of the clusters, with higher values indicating better quality. When both subsets yielded the same number of clusters, a predictive discriminant analysis (using R's MASS package) classified test cases based on the training-data cluster solution, with a target of 75% classification accuracy to ensure reliable stability (Lepp et al., 2015). Finally, both subsets were recombined and analyzed to generate the "best" final cluster solution.

For further exploration, heat map visualizations (Wilkinson & Friendly, 2009) were used to examine variations across grade levels and function types, providing additional insights into the diversity of strategies employed in creating function-based art.

3.4.3. Qualitative Analysis

For qualitative analysis, we selected one representative from each cluster and analyzed them using the rubric derived from the TIMSS Framework. The goal of the analysis is to delve deeper into the students' artwork. In our analysis, we used the rubric in Table 2, derived from the TIMSS Framework and Bautista, Soeharto, et al. (2025), to classify the use of functions, precision, and strategies in the artwork into Knowing, Applying, and Reasoning. Although the TIMSS framework is broadly applicable across various mathematical domains, we adapted it to focus specifically on student engagement with functions in the context of function art. We acknowledge, however, that the TIMSS cognitive framework was originally developed for structured, test-based assessments rather than for open-ended, creative tasks. Our use of the Knowing, Applying, and Reasoning domains is an adaptation meant to serve as an interpretive lens for examining patterns in students' mathematical thinking. This approach builds on emerging research that explores the use of TIMSS and similar frameworks for cognitive diagnosis beyond standardized assessments (e.g., Lee et al., 2011). While the framework has not been formally validated for this context, we employed clear operational criteria (see Table 2) and achieved high inter-rater reliability ($\kappa = 0.95$), indicating consistent application. Nonetheless, we recommend further validation of such adaptations in future studies focused on interdisciplinary and open-ended STEAM tasks. The evaluation was structured around three metrics, as shown in Table 2.

Table 2. The Rubric for Analyzing and Classifying Function Art Based on the TIMSS Framework

Components	Knowing	Applying	Reasoning
M1: Use of functions	10 functions or less	11–20 functions	21 functions or more
M2: Precision on the use of transformation, symmetry, and domain	more than 5 imprecise transformation, symmetry, or definition of domain	3–5 or more imprecise transformation, symmetry, or definition of domain	2 or less imprecise transformation, symmetry, or definition of domain
M3: Use of Strategy	0 strategy used	1 strategy used	2 or more strategies used

M1: Number of Functions – This metric reflects students' frequency of functions. A higher function count suggests greater proficiency. Artworks were categorized as Knowing (≤ 10 functions), Applying (11–20 functions), or Reasoning (≥ 21 functions).

M2: Precision in Transformations, Symmetry, and Domain Use – This metric evaluates technical accuracy in applying function transformations and defining domains. Key criteria included:

Use of algebraic structure in patterned transformations (e.g., forms such as $g(x) + nk$ or $g(x - nh)$),

Proper alignment of function segments at junction points (e.g., domains of adjacent functions sharing endpoints: $[a, b]$ and $[b, c]$),

Correct domain definitions for symmetric functions (e.g., domains of the form $[-c, c]$ for y-axis symmetry),

Accurate reflections, where both equation and domain reflect the intended transformation (e.g., reflecting across the y-axis requires appropriate sign changes in both).

Artworks with fewer technical errors were rated more highly.

M3: Strategy Use in Function Art – This metric assessed the sophistication of techniques used, based on strategies identified by Bautista, Soeharto, et al. (2025), including:

Converting conic sections (e.g., circles) into functions using piecewise representations,

Using inequalities to shade regions,

Applying symbolic transformations such as $f(x)$, $-f(x)$, and $f(-x)$,

Using sliders for dynamic curve generation,

Designing algorithms or tools (e.g., using the Point tool for efficient coordinate extraction).

Artworks were classified as Recall (no strategies used), Apply (one strategy), or Reason (two or more strategies).

Artwork that does not incorporate any of these strategies is classified under Recall, artwork using one strategy under Apply, and artwork using two or more strategies under Reason.

To support transparency in our mixed-methods approach, Table 3 summarizes the analytical categories used in the study, including their definitions and examples.

Table 3. Coding Criteria, Indicators, and Examples

Analytical Category	Definition/Coding Criteria	Aim in the Integrated Analysis	Coding Indicators	Illustrative Example from Data
Function Use and Diversity	Quantitative measure of the number and variety of mathematical functions (linear, quadratic, trigonometric, etc.) used in each artwork.	To identify the *breadth and variation* in students' math–art cognitive strategies and provide the quantitative basis for clustering learner profiles.	Count of total functions per artwork; count of unique function types.	"The student used multiple sine and quadratic functions to produce layered wave patterns."
Transformation Strategy	How students applied transformations (translations, dilations, reflections, domain restrictions) to create visual or structural patterns.	To reveal students' *procedural fluency and experimentation* in manipulating mathematical functions within an artistic context.	Frequency and precision of transformations; evidence of systematic parameter manipulation.	"The student systematically shifted parabolas along the *x*-axis to form arches."
Representational Complexity	Degree of integration between mathematical and visual representations, indicating conceptual depth and aesthetic synthesis.	To capture the *conceptual and aesthetic sophistication* of how mathematical ideas are represented through art.	Presence of multiple function types interacting meaningfully; composition coherence and symmetry.	"Different function families were combined symmetrically to create a butterfly-like figure."
TIMSS Cognitive Domain	Classification of cognitive engagement using adapted TIMSS domains: *Knowing*, *Applying*, *Reasoning*. Used to interpret conceptual sophistication in function use.	To interpret *levels of mathematical reasoning* within artworks and connect artistic processes to recognized cognitive frameworks.	Knowing: Recall and basic identification. Applying: Correct procedural execution. Reasoning: Generalization or creative synthesis.	"The student identified basic functions (Knowing), adjusted parameters for symmetry (Applying), and explained transformations conceptually (Reasoning)."
Learner Profile (Cluster)	Cluster membership based on HCPC of function-use variables, reflecting patterns of engagement.	To synthesize quantitative and qualitative findings into *distinct learner profiles* that represent different cognitive and representational strategies.	Cluster 1 = *Simplists*; Cluster 2 = *Repetitivists*; Cluster 3 = *Multifunctionists*.	**Simplists:** minimal exploration, low quantity/diversity. **Repetitivists:** many similar functions, limited variation. **Multifunctionists:** diverse function use with conceptual experimentation.

These categories informed both the clustering process and the interpretation of students' cognitive and representational strategies.

4. Results

In what follows, we present an integrated result of how students engaged within the context of function art, organized to illuminate how distinct learner profiles—emerging from both quantitative and qualitative patterns—embody different cognitive and representational strategies. We begin with a quantitative analysis of the types and numbers of functions used across students' artwork, followed by a clustering analysis that reveals three distinct learner profiles. To further illustrate the epistemic and expressive characteristics of each cluster, we conclude with a qualitative examination of representative student creations.

Table 4 presents the mean and standard deviation for both the number of functions (MF, SF) and the number of function types (MT, ST) used, disaggregated by grade level. Because the original data exhibited high variance and skewness—particularly in the number of functions used—we applied a log-transformation (log1p) to improve interpretability and reduce the influence of extreme values. The resulting descriptive statistics provide a clearer view of trends across grades.

As shown in Table 4, Grade 8 exhibits the highest average number of functions used (M log F = 3.96, S log F = 0.96), while Grade 10 shows the lowest mean (M log F = 2.62, SF = 0.84), with Grades 11 and 12 remaining relatively stable and slightly higher. Despite the transformation, variation remains relatively high across all grades, indicating diverse strategies in how students incorporated functions into their work.

Interestingly, the mean number of function types (M log T) remains relatively stable across grade levels. Grade 8 has a higher M log T (0.99) than Grade 9 (0.88), even though Grade 9 students are typically exposed to more types of functions. Grades 10, 11, and 12 exhibit closely aligned M log T values (~1.09–1.17).

Table 4. Mean and Standard Deviation of the Number/Types of Functions by Grade Level

Grade Level	N	MF **(M log F)**	SF **(S log F)**	MT **(M log T)**	ST **(S log T)**
Grade 8	79	73.75 (3.96)	51.24 (0.96)	1.90 (0.99)	1.37 (0.38)
Grade 9	77	56.64 (3.51)	52.20 (1.18)	1.83 (0.88)	0.62 (0.31)
Grade 10	57	20.19 (2.62)	26.04 (0.84)	2.35 (1.09)	0.58 (0.33)
Grade 11	144	21.13 (2.68)	22.94 (0.86)	2.34 (1.16)	0.56 (0.39)
Grade 12	43	17.74 (2.70)	17.35 (0.75)	2.33 (1.17)	0.52 (0.39)

Note: Log-transformed means and standard deviations are shown in parentheses.

4.1. Cluster Analysis Results

To investigate how students approached mathematical functions through artistic creation, we conducted cluster analyses to identify distinct learner profiles. Prior to clustering, the two key variables—the number of functions used and the number of function types—were log-transformed to reduce skewness and then standardized using z-scores to place them on the same scale. These standardized variables served as inputs for both HCPC and k-means clustering. Clustering was performed on a training dataset using both methods, resulting in a consistent three-cluster solution. A good cluster solution should have cohesive elements within clusters and clear separation between them (Aldenderfer & Blashfield, 1984; Everitt, 1979). The Silhouette coefficient of .30 in both the training and test datasets (analyzed independently) indicated fair quality, and the consistent three-cluster outcome in each dataset suggested that the clusters served the same purpose (Lepp et al., 2015).

A predictive discriminant analysis was then employed on the training dataset, using the cluster solution as the dependent variable and two dimensions of function art (number of functions and type of functions) as independent variables. This generated a linear classification rule, which was subsequently applied to the test dataset to predict cluster membership. Results demonstrated that the classification accurately assigned about 94.4% of cases to their correct cluster, confirming strong external validity. Fewer than 10% of observations were reclassified (Hair et al., 2006), indicating stable cluster patterns across independent samples and thereby validating the original cluster solution.

When performing Cluster analysis on the whole dataset, three clusters emerged. Cluster 1: "Repetitivist" (n = 62, 15.5%, 19 males, 43 females): students with a low number of function types and a high number of functions. Cluster 2: "Simplist" (n = 226, 56.5%, 88 males, 138 females): students with a low number of function types and a low number of functions. Cluster 3: "Multifunctionist" (n = 112, 28%, 36 males, 76 females): students in this cluster performed high on the number of function types and lowest on the number of functions. These profiles align with principles of cognitive constructivism, which view learning as an active process involving individual construction of knowledge, progression through stages of abstraction, and the use of varied representational strategies (Clements & Battista, 1992; Papert, 1991). High function counts reflect iterative knowledge building, while greater function diversity indicates flexible and abstract reasoning. Thus, the clusters are not only data-driven but also theoretically grounded, offering insight into students' cognitive strategies during math–art integration.

To evaluate the differences between groups, MANOVAs were conducted. Since n was unequal for the cluster groups, we conducted Welch's ANOVA to compare cluster variables. Welch's ANOVA is an adjustment to the ANOVA for independent samples because it does not assume equal population variances, while retaining the assumption of normality (Liu, 2015). Table 5 displays the significance level for each cluster. Figure 3 shows the distribution of Mean values of clustering variables across clusters' profiles.

Table 5. Means, Standard Deviations, and Welch's F Test Values of Clustering Variables Across Cluster Profiles

Cluster profiles	Cluster 1 (n = 62) *Repetitivist*	Cluster 2 (n = 226) *Simplist*	Cluster 3 (n = 112) *Multifunctionist*	Welch F
ZNo_of_functions	1.925 (0.809)	-0.4 (0.419)	-0.258 (0.642)	432.914**
ZNo_of_Types	-0.399 (0.641)	-0.516 (0.361)	1.262 (0.929)	328.597**

*Note: Mean values are standardized (z) scores with standard deviations in parentheses. **p <.01.*

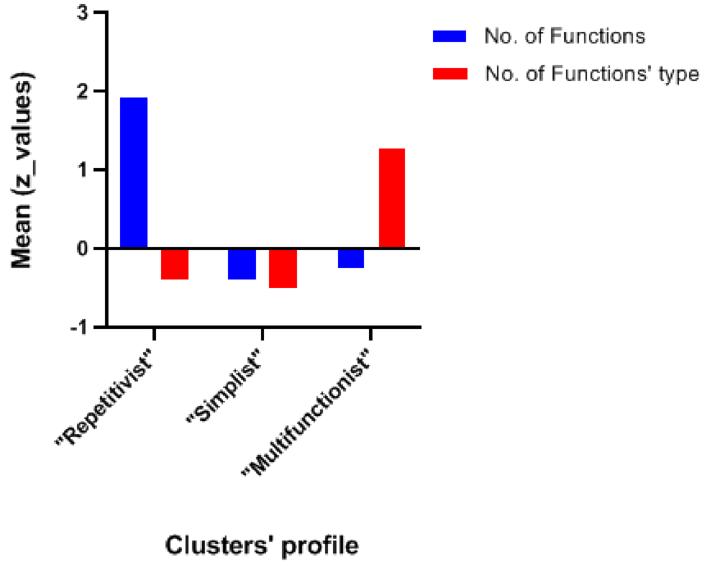

Figure 3. Students' Cluster Profile

In the subsequent stage of the analysis, a series of chi-square tests of independence were conducted to examine the relationship between cluster membership and gender, grade level, and function art type. The results indicated that gender was not significantly associated with cluster membership, $\chi^2(2) = 2.33$, $p = .31$, Cramér's V = 0.07, suggesting that the distribution of gender did not differ across the three clusters. In contrast, significant associations were observed for both grade level and function type. Specifically, the chi-square test for grade level was significant, $\chi2(8) = 79.32$, $p < .001$, Cramér's V = 0.31, as was the test for function type, $\chi2(4) = 20.18$, $p < .001$, Cramér's V = 0.15.

To further investigate these significant associations, post hoc comparisons were conducted using adjusted standardized residuals to control for multiple comparisons, and results were visualized using heat maps. In terms of grade level (Figure 4), the adjusted residuals revealed that the Repetitivist group included a significantly higher proportion of lower-grade students—specifically Grades 8 and 9—compared to the Simplist and Multifunctionist groups (adjusted residuals > 1.96, p <.05). In contrast, the Multifunctionist group showed a significantly greater representation of upper-grade students—namely Grades 10 and 11—relative to the other two groups. Similarly, regarding function art type (Figure 5), post hoc comparisons indicated that the Repetitivist group significantly differed from both the Simplist and Multifunctionist groups in relation to the use of the Pure function art type, with adjusted residuals exceeding the critical value (adjusted residual > 1.96, p <.05). No other pairwise comparisons yielded statistically significant differences.

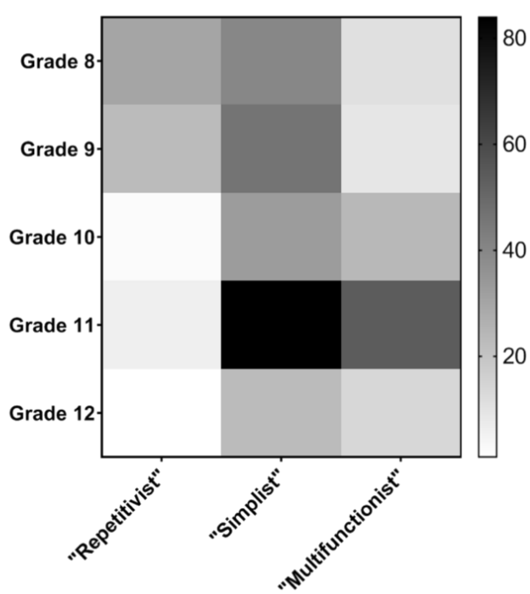

Figure 4. Heat Map of Cluster Versus Grade Level

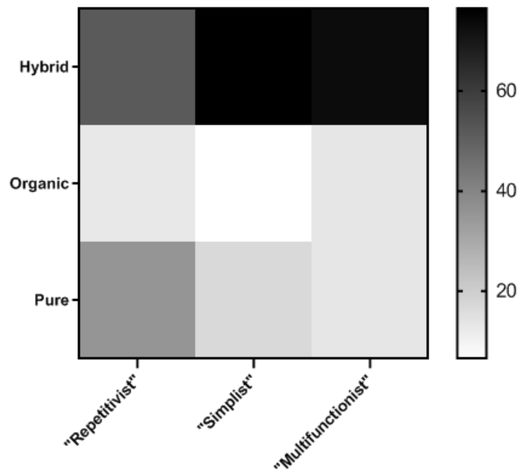

Figure 5. Heat Map of Clusters Versus Types of Function Art

To determine whether cluster membership (Multifunctionist, Repetitivist, Simplist) corresponded to distinct patterns in the use of various mathematical functions during the function art activity, Welch's ANOVA was conducted. As shown in Table 6, the analysis revealed no significant differences among the three cluster groups in their average usage of logarithmic (log), exponential (exp), radical (rad), or other (otf) functions during the function art activity. However, significant differences were observed for sine (sin), cosine (cos), linear (lin), absolute value (abs), quadratic (qua), and polynomial (pol) functions, indicating that cluster membership influenced the frequency with which these function types were utilized.

Follow-up Games-Howell post hoc tests indicated that Multifunctionist students employed sine, cosine, quadratic, and polynomial functions significantly more often than Simplist students, and employed absolute value functions significantly more often than Repetitivist students. In contrast, Simplist students used linear functions significantly more often than Multifunctionist students, whereas Repetitivist students used linear functions significantly more often than both Simplist and Multifunctionist students. These findings suggest that the Multifunctionist group exhibited a broader range of function usage, while the Simplist and Repetitivist groups showed relatively higher frequencies of specific function types.

Table 6. Welch's ANOVA Results and Games–Howell Post Hoc Comparisons Among the Three Cluster Groups

Cluster comparison variables	Repetitivist M(SD)	Simplist M(SD)	Multifunctionist M(SD)	Welch F	Games–Howell post hoc multiple comparison tests
Zscore(SIN_COS)	.232(2.3)	-.127(.4	.129(.5)	11.4**	Cluster3 reported significantly higher numbers than Cluster 2
Zscore(EXP_LOG)	.199(2.3)	-.068 (.1)	.027(.5)	1.75	No significant differences
Zscore(CON)	.115(2)	-.057(.7)	.052(.5)	1.31	No significant differences
Zscore(LIN)	1.76(1.33)	-.282(.47)	-.407(.42)	80.44**	Cluster 1 reported significantly higher numbers than Cluster 2 and Cluster 3 Cluster 2 reported significantly higher numbers than Cluster 3
Zscore(ABS)	-.123(.16)	-.078(.93)	.227(1.40)	3.57**	Cluster 3 reported significantly higher numbers than Cluster 1 No significant differences between Cluster 2 and Cluster 3 3>2>1
Zscore(QUA)	.221(1.89)	-.12(.61)	.121(1.07)	3.72**	Cluster 3 reported significantly higher numbers than Cluster 2
Zscore(POL)	.139(1.86)	-.114(.85)	.15(.45)	3.82**	Cluster 3 reported significantly higher numbers than Cluster 2
Zscore(RAD)	.329(2.34)	-.07(.53)	-.03(.38)	1.187	No significant differences
Zscore(Other)	.04 (1.33)	-.02(.82)	.06(1.19)	0.283	No significant differences

*Note: Mean values (M) are standardized (z) scores with standard deviations (SD) in parentheses. Function types are coded as follows: sine and cosine (SIN_COS), exponential and logarithmic (EXP_LOG), constant (CON), linear (LIN), absolute value (ABS), quadratic (QUA), polynomial of degree three and above (POL), and radical (RAD). **p <.01.*

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4.2. Qualitative Analysis Results

In this section, we discuss one sample artwork from each cluster. The qualitative examples provided in the following sections are intended to serve as illustrative case studies rather than a statistically representative sample. They have been carefully selected to highlight the defining characteristics and complexities of each student cluster. The selections were made based on specific criteria that included interesting mathematical properties, such as the conversion of linear equations and the application of symmetry, the number and strategic use of functions, and the overall aesthetic quality of the resulting artwork. By focusing on these specific instances, we aim to provide a deeper, qualitative understanding of the strategies students employ in each cluster, complementing the quantitative analysis presented in the preceding section.

4.2.1. Sample Repetitivist (Cluster 1) Art

Figure 6 presents an artwork from the Repetitivist cluster, a pure function art. This cluster of artworks is characterized by a high number of functions but a low number of function types. This fox artwork includes 93 functions in total, consisting of 91 linear functions and two constant functions, and therefore classified as Reasoning under M1.

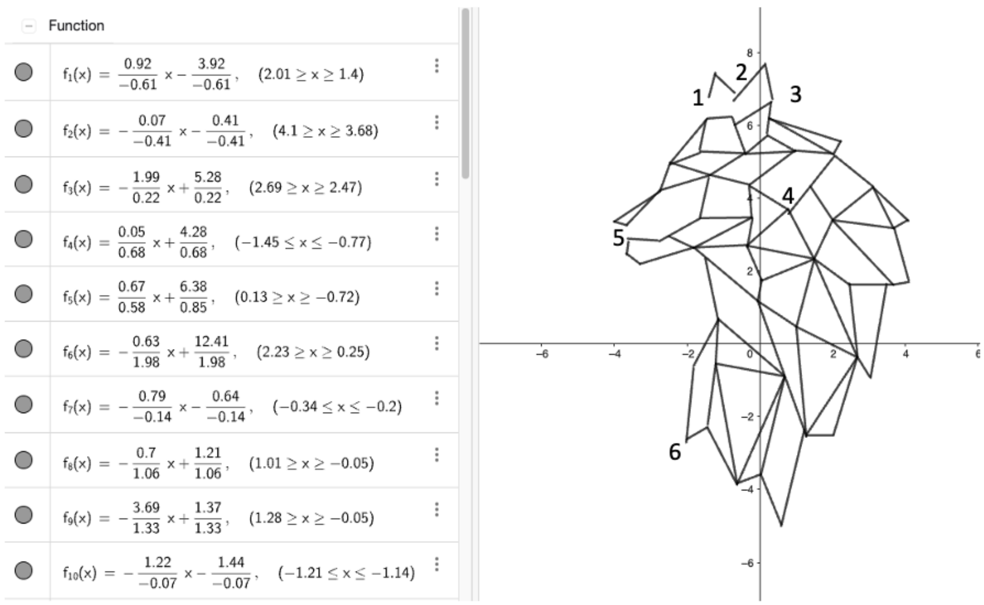

Figure 6. Sample of Repetitivist Artwork

For M2, it is visually clear that more than five junction points are not precisely connected, as shown by the numbers in Figure 6. Therefore, we classify this as Knowing because of the imprecise domains.

Grade 8 students are familiar with the slope-intercept form $f(x) = mx + p$ of linear functions. However, finding the values of m and p for each of the 91 functions can be time-consuming. To streamline the process, the student used the Line tool to draw each line. This tool automatically generates three objects: the two points through which the line passes and the line itself. In the Algebra view, GeoGebra provides the coordinates of these points as well as the general equation of the line in the form $ax + by = c$. The student transformed the general equation into the slope-intercept form $f(x) = (a/b)x + c/b$, used the points to determine the endpoints of the curve, and then adjusted the *domain* using their x-coordinates. Note that during the transformation, she did not calculate a/b and c/b; she just typed the fraction. For instance, the line f_1 in Figure 6 has the original equation of $-0.92x - 0.61y = -3.92$. The student changed it to

$$f_1(x) = \frac{0.92}{-0.61}x - \frac{3.92}{-0.61}$$

with domain [1.4, 2.01]. In this scenario, the student devised an algorithm to speed up the construction of the artwork, one of the strategies identified in M3. Therefore, we classified this as Applying.

4.2.2. Sample Simplist (Cluster 2) Art

Figure 7 is an example of an artwork from the Simplist cluster, a hybrid function art. This cluster of artworks is characterized by low number of functions and low number of function types. The artwork contains eight functions and two function types, linear and quadratic.

For M1, there are only eight functions, so we classify it as Knowing. For M2, the transformation includes reflection about the y-axis. All the parameters of the pair of functions of the form $p(x) = a(x - h)^2 + k$ are precise. For instance, the pair f and i

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have equations $f(x) = 0.7(x - 2)^2 + 3.7$ and $i(x) = 0.7(x + 2)^2 + 3.7$, respectively. These functions are precisely the reflection of each other across the *y*-axis, with pairs of parameters *a* and *k* equal, and *h,* which are additive inverses of each other.

In addition, regarding the domain, the student defined it precisely. For the gluing point of *i* and *f,* for example, the right endpoint of *i* is at $x = 0$, while the left endpoint of *f* is also at $x = 0$. For the reflection about the *y*-axis, the domain of the curves *i* and *f*, [-7.34, -5.99] and [5.99, 7.34] were defined precisely. Therefore, in terms of M2, we classify this as Reasoning.

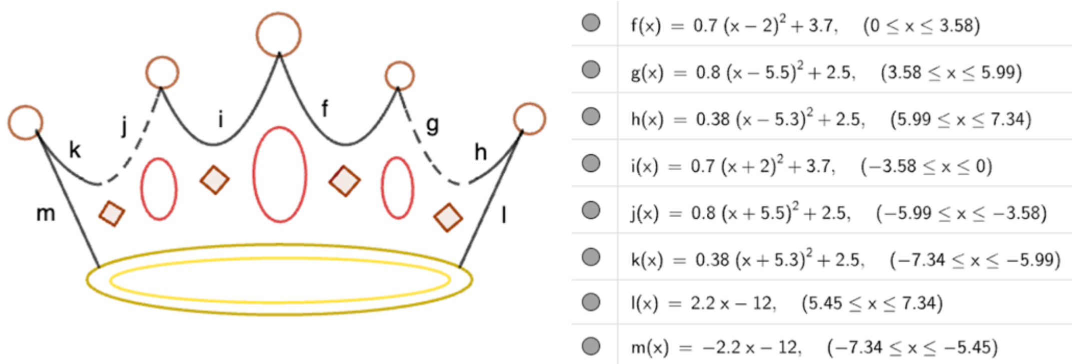

Figure 7. Sample of a Simplist Artwork

For M3, none of the strategies were used, so we classify it as Knowing. The most appropriate strategy here is to use symbols to reflect across the *y*-axis. The student can graph the function $f(x)$ on the domain $[a, b]$ and then reflect it across the *y*-axis using the notation $f(-x)$.

We did not compute inter-coder reliability for M2 (precision) and M3 (strategy use) because these were applied only to a small set of sample artworks in the qualitative analysis. Both categories involve detailed mathematical interpretation and were used illustratively rather than for generalization. Coding every artwork was not feasible due to the time and complexity involved. For M2, we followed clear operational criteria to ensure consistency. As for M3, a separate paper focusing specifically on strategy use is currently in preparation, which will include a more comprehensive analysis and inter-coder reliability.

4.2.3. Sample Multifunctionist (Cluster 3) Art

Figure 8 is an example of an artwork from the Multifunctionist cluster, an organic function art. This cluster of artworks is characterized by a low number of functions and a high number of function types. This artwork features 29 functions, so, for M1, we classified it as Reasoning. Note that in the TIMSS framework, 29 is classified as Reasoning because it is the average number of functions for the entire data set.

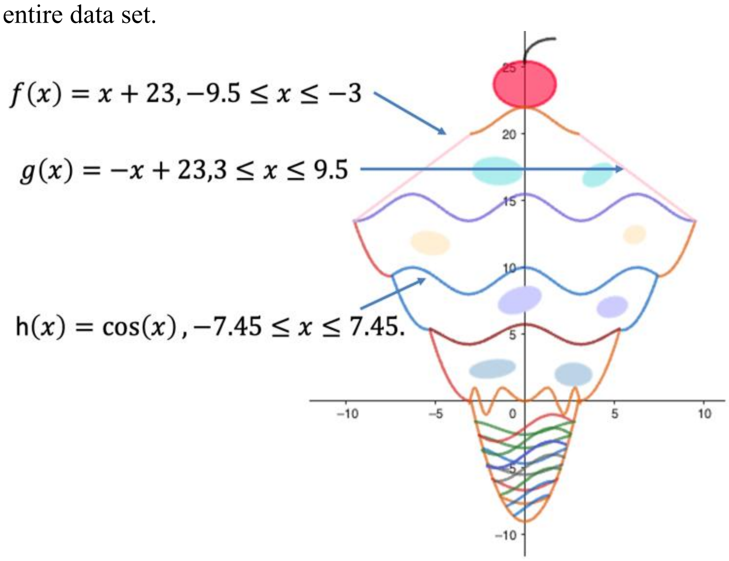

Figure 8. Sample of a Multifunctionist Artwork

For M2, looking at the equations, the curves are precise. For pairs of functions that are reflections of each other across the *y*-axis, their equations and domains are both precise. For example, $g(x)$ is the exact reflection of $f(x)$ about the *y*-axis. In addition, the functions which are symmetric about the *y*-axis have precise domains of the form $[-c, c]$. For M3, none of the strategies were used, so we classified it as Knowing. Just as in the Simplist example, the use of the symbols $f(x)$ and $f(-x)$ could have expedited the creation process.

5. Discussion

In this section, we delve into the implications and significance of our study's findings for function art. Our research aimed to investigate the extent of students' use of functions and their strategies for using them to create their artwork, and its implications for the transdisciplinary nature of function art.

The findings of the cluster analysis highlight distinct patterns in students' engagement with function types and their artwork, which can be linked to the TIMSS framework. TIMSS evaluates students' mathematical achievement by considering cognitive domains (Knowing, Applying, and Reasoning) and content domains (Algebra, Number, Geometry, and Data) (Mullis & Martin, 2017). The differences observed among the three clusters—Repetitivist, Simplist, and Multifunctionist—can be interpreted through these domains.

5.1. Cognitive Strategies and Function Art Cluster Membership

The cognitive distinctions identified among the three clusters reflect well-established theories of how mathematical understanding evolves through increasing levels of abstraction and representational fluency. TIMSS emphasizes that mathematical reasoning and problem-solving form the foundation of high-level proficiency (Mullis & Martin, 2017, 2019). The Multifunctionist cluster—students who employed a diverse range of function types, including trigonometric and polynomial—exemplifies the reasoning domain described in TIMSS (Mullis et al., 2020). Their ability to flexibly connect and apply mathematical structures aligns with cognitive constructivist perspectives, which posit that knowledge is built through active engagement and reflection on multiple representations (Gavrilas & Kotsis, 2025). Consistent with cognitive theories of abstraction (Schneider & Stern, 2010), these learners demonstrate integrative problem-solving, coordinating the symbolic, graphical, and contextual dimensions of functions to construct deep conceptual networks.

Empirical evidence from STEAM research supports these cognitive patterns. Studies have shown that embedding mathematical ideas in creative, multimodal contexts—such as function art—enhances metacognition, conceptual understanding, and visual reasoning. Wahba et al. (2022) reported that participation in STEAM-based mathematics tasks increased students' metacognitive awareness and self-regulation, both of which underpin conceptual transfer. Similarly, Thuneberg et al. (2018) found that creative autonomy and visual reasoning in STEAM activities fostered abstract thinking and flexible problem-solving strategies. These findings resonate with the cognitive characteristics of the Multifunctionist cluster, suggesting that art-integrated mathematical experiences cultivate the same higher-order reasoning and imaginative engagement required for expert mathematical performance.

By contrast, Repetitivist students relied heavily on a limited set of functions, most commonly linear functions. This pattern suggests rule-based strategies that focus on algorithmic application without adaptation to unfamiliar scenarios (Lemaire & Reder, 1999). Their approach underscores procedural fluency, centred on executing known operations rather than exploring the functions' inherent properties. Such behaviour aligns with TIMSS's Knowing domain, which entails recalling and straightforwardly Applying mathematical formulas (Mullis & Martin, 2019). This group's preference for linear functions may indicate a lower inclination to generalize or engage with more complex problem-solving processes.

Simplist students exhibited both low function diversity and low function frequency. Their tendency to use only the most elementary operations reflects a cognitive avoidance strategy (Efklides, 2011) and suggests surface-level processing (Marton & Säljö, 1976). In TIMSS terminology, these students appear to invest minimal effort in the Applying domain, attempting direct and simple solutions without deep analytical engagement (Mullis et al., 2020). Their limited range of function use may also relate to cognitive load issues (Sweller, 1994), in which the complexity of higher-order functions leads to avoidance of the more demanding aspects of mathematical representation and creativity.

From an instructional and LA perspective, these findings suggest that cluster-based profiling can inform adaptive, evidence-driven pedagogy. LA can be used to identify students' cognitive profiles—such as their functional diversity, representational use, and reasoning indicators—and to tailor STEAM interventions accordingly.

5.2. Grade-Level Differences and TIMSS Curriculum Expectations

Grade-level variations in mathematical reasoning are frequently interpreted through frameworks such as TIMSS, which anticipate a developmental progression from concrete toward increasingly abstract modes of problem solving (Mullis et al., 2020; Peters, 2024). Consistent with these expectations, our findings indicate that older students (Grades 10–11) engaged with a broader repertoire of function types and exhibited greater conceptual flexibility than younger peers (Grades 8–9). Typically, Multifunctionists—predominantly older students—explored polynomial and trigonometric models, while Simplists and Repetitivists—more common among younger cohorts—tended to rely on procedural or computational approaches (Mullis &

Martin, 2019). These distinctions mirror TIMSS's characterization of cognitive maturation from procedural fluency toward abstract generalization.

Yet, the data-driven clustering of students' function-based artworks revealed cognitive dynamics that extend beyond conventional curricular progressions. Situated within a transdisciplinary STEAM framework, the activities encouraged learning that "transcends the boundaries of particular disciplines" (Mejias et al., 2021; Costantino, 2018). Although cluster membership broadly aligned with grade level, notable exceptions emerged: some younger students demonstrated advanced transformations, while a few older students relied on repetitive strategies. Such patterns indicate that math–art integration can foster abstract reasoning irrespective of grade, expanding opportunities for conceptual growth across levels.

This cross-grade potential highlights how mathematical and artistic reasoning can operate as mutually reinforcing processes. In our tasks, mathematical understanding and artistic expression were interdependent, requiring students to engage both conceptually and creatively. As Mejias et al. (2021) argue, effective STEAM environments must navigate tensions between artistic and scientific epistemologies without privileging one over the other. Our findings illustrate this reciprocity: rather than treating art as an aesthetic supplement to mathematics, the function-art approach positions both as co-constructive domains that open new pathways to reasoning.

Recent research supports this interpretation. Calderón-Tena (2016) contends that mathematical development relies on broad cognitive processes—such as abstraction, representation, and transfer—rather than on content accumulation alone. Our results align with this view: function-art activities prompted students to coordinate analytical and creative cognition, stimulating the kind of higher-order reasoning typically associated with later stages of mathematical development. Similarly, Costantino (2018) emphasizes that transdisciplinary art and design practices cultivate flexible, integrative thinking, as evidenced by students' ability to approach functions through visual and expressive modalities. Complementing these perspectives, Sylviani et al. (2024) demonstrate that integrating visual arts into mathematics enhances students' engagement and interest by providing aesthetic and affective entry points into abstract ideas.

Taken together, the three clusters—Repetitivist, Simplist, and Multifunctionist—reflect both TIMSS's model of developmental progression and the transformative potential of art–math integration. Function-art tasks can elicit conceptual engagement that crosses grade boundaries, aligning with international benchmarks while advancing transdisciplinary and interest-driven learning.

5.3. Sample Artwork from Each Cluster

The fox artwork from the Repetitivist cluster illustrates a student's ability to optimize workflow, even when using only linear and constant functions. Instead of manually calculating slopes and intercepts, the student used GeoGebra's Line tool to generate equations, making the method faster and more efficient. This approach aligns with the Applying category in M3, as the student implemented a strategic method to simplify the process. However, imprecise domain restrictions place aspects of their work in the Knowing category in M2.

The predominance of younger students in this cluster suggests that their knowledge of functions is limited by their curriculum, leading to repetition of function types. However, their problem-solving approach highlights early signs of mathematical reasoning. This finding aligns with prior research on students' early encounters with functions, which often reveal a reliance on familiar, recently learned function types such as linear functions. Studies have shown that younger students or those newly introduced to functions tend to engage in function mimicry—repeating similar forms due to limited exposure and conceptual understanding (Confrey & Smith, 1995). This repetition may reflect both cognitive limitations and the structure of early algebra curricula, where linearity is a central focus (Blanton et al., 2019). Moreover, the emergence of early signs of mathematical reasoning, even within repetitive patterns, aligns with emergent algebraic thinking, in which learners begin to explore structure and relationships through pattern generalization and symbolic representation (Radford, 2006). In the context of function art, the constrained choices of younger learners become a productive space for developing algorithmic thinking—the ability to plan, sequence, and execute rule-based graphing tasks (Zhang & Jia, 2024).

The use of straight lines and a preference for linearity in function art, especially in the Repetitivist cluster, reflect what Papert (1980) described as low-threshold, high-ceiling affordances: linear functions are accessible to beginners yet can be recombined creatively. This suggests a direct correlation between students' current mathematical knowledge and their creative outputs. As students progress and are introduced to more complex functions, such as quadratics or trigonometric functions, their artistic expressions are likely to evolve, incorporating a broader range of mathematical concepts. This progression not only enriches their creative endeavours but also deepens their understanding of mathematical relationships and functions.

In cases where curves were desired but not easily represented with functions, the use of geometric tools suggests an evolving understanding of mathematical representation. This mirrors findings by Yerushalmy (1997), who noted that students often shift between visual and symbolic representations when they encounter limitations in one modality.

The crown artwork in the Simplist cluster shows how a student focused on precision rather than complexity. The student used only eight functions and two function types, which fall under Knowing in M1. However, all of the transformations and junction points are precise, which falls under Reasoning in M2. The students in this cluster used fewer functions, which may have given them more time to be precise. They are the students who prefer quality over quantity in creating their artwork. This matches patterns observed in prior research (e.g., Stylianou, 2002; Leinhardt et al., 1990), which discuss how expert-like behaviour can manifest in minimalist approaches, in which students optimize accuracy and cohesion with limited resources.

The ice cream artwork from the Multifunctionist cluster shows how a student used fewer functions but many types of functions. The student used a mix of linear, quadratic, and trigonometric functions to create smooth curves. This work falls under Reasoning for function use and Applying for precision, as the symmetry and reflections are correct, but not always optimized. This aligns with the work of Yerushalmy (1997), who emphasized the role of visual intuition and experimentation in mathematical thinking, particularly in dynamic geometry or function-graphing environments.

The classification of functions using the TIMSS Framework indicates that the number and types of functions are insufficient to determine the mathematical quality of students' artwork. This means that the teacher must delve deeper into the functions students use and the strategies they employ to use those functions. For instance, in the Fox artwork from Cluster 1, although the student used mainly linear functions, they developed an efficient method for graphing straight-line segments. Therefore, we can conclude that the student understands the algebraic relationship between the form $ax + by = c$ and $f(x) = (a/b)x + c/b$. On the other hand, due to the imprecise gluing points, we can hypothesize that the student still lacks an understanding of the domain of functions.

Additionally, in the Crown artwork, even though the student used only two types of functions, mainly quadratic functions, they demonstrated their knowledge of function transformations and domain by applying precise transformations and intervals.

5.4. Limitations

The study has several limitations that should be considered when interpreting the results. First, the participating teachers were recruited through convenience sampling, which introduces self-selection bias. The teacher's familiarity with the first author may have influenced their willingness to participate in the project and possibly their engagement with the students. Second, there was no baseline measure of students' understanding of functions, which may have affected their engagement with function art activities. Third, although specific functions were introduced as examples for each grade level, most instructional resources were similar despite their differences in curricular exposure and developmental readiness. Future iterations should develop differentiated materials tailored to specific grade levels.

In addition, the study only analyzed students' final artworks. Even though the functions and completed steps can be examined, they are already in their final form and do not show the intermediate steps in students' problem-solving processes. For future studies, screen recording could be used to document the step-by-step work and provide deeper insights into their thinking.

Lastly, the detailed rubric used to assess function art may be too complex for practical classroom use. We recommend simplifying the assessment by retaining components M1 and M3, and replacing M2 with a focus on the artwork's aesthetic beauty.

5.5. Implications

This study provides several implications for classroom practice and education policy. For practitioners, the findings show that function art can be used not just as an enrichment activity, but as a tool for exploring students' mathematical thinking across different cognitive levels. Teachers can use function art tasks to surface students' understanding of transformations, domains, and symbolic manipulation, while also promoting creativity and student agency. The use of the TIMSS cognitive domains as an assessment lens offers a practical, research-based way to evaluate students' mathematical engagement in open-ended tasks.

In practice, educators can differentiate instruction by paying attention to student profiles—Repetitivists, Simplists, and Multifunctionists—and designing support that meets each group's strengths and needs. For example, students who tend to repeat function types may benefit from tasks that prompt them to experiment with new forms or reflect on their strategy use.

For policymakers and curriculum designers, the results support the inclusion of interdisciplinary tasks such as function art in formal mathematics curricula. While STEAM is often promoted in general terms, this study provides concrete evidence that integrating mathematics and visual arts can deepen students' conceptual understanding. Policies that encourage creative, technology-supported tasks—especially those that bridge disciplines—can help foster the kind of flexible, higher-order thinking that modern education systems value. Additionally, professional development should emphasize how tools such as GeoGebra and frameworks such as TIMSS can be adapted to assess creative mathematical performance in real classrooms.

6. Conclusions

This study demonstrates the potential of function art as both an expressive and analytical tool for exploring students' mathematical understanding within a STEAM context. Using clustering techniques and a mixed-methods approach grounded in the TIMSS framework, we identified three learner profiles that reveal how students conceptualize and represent functions differently across grade levels and cognitive strategies. While Repetitivists emphasize quantity through familiar function types, Simplists value precision over complexity, and Multifunctionists integrate a diverse array of functions with moderate frequency—reflecting more advanced conceptual reasoning. Importantly, the study underscores that the number and type of functions alone do not fully capture the depth of mathematical thinking; rather, the strategies and precision students employ provide richer insight into their cognitive engagement. These findings highlight the importance of supporting diverse learning pathways and integrating function art meaningfully in curriculum design. Future research may expand on this work by incorporating longitudinal data and examining the development of students' mathematical reasoning over time through similar interdisciplinary approaches.

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